Indian Statistical Institute, Bangalore
B. Math. First Year, First Semester
Probability Theory: Mid-Term Examination

Date : 10-09-2014 Time: 3 hours
Maximum score: 100

1. There are balls numbered $1, 2, \ldots, 8$ in a box. Odd numbered balls are colored black and even numbered balls are colored white. Go on choosing balls at random from the box, without replacement, until you get two balls of the same color. After this experiment, let M be the number of balls remaining in the box. Find the probability distribution of M. Write down probability mass function and probability distribution functions of M. [20]

2. Let W be a random variable with distribution function F given by

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ \frac{2}{5} & \text{if } 1 \leq x < 2 \\ \frac{3}{4} & \text{if } 2 \leq x < 5 \\ 1 & \text{if } 5 \leq x < \infty \end{cases}$$

Compute the following probabilities: (a) $P(W \leq \frac{2}{5})$; (b) $P(\frac{2}{5} \leq W < \frac{5}{2})$. Write down the distribution function of $2 - W$. [20]

3. There are two bags. The first bag has 10 fifty paise coins and 20 one rupee coins. The second bag has 5 fifty paise coins, 15 one rupee coins and 15 two rupee coins. One bag was chosen at random and from that bag one coin was chosen at random. Given that the coin chosen was a fifty paise coin, what is the conditional probability that it is from the first bag? [10]

[P.T.O.]

4. Let Z be a Poisson random variable with parameter $\lambda > 0$. Find expectation and variance of Z. Find expectation and variance of $V = 2Z - 5$.

[20]

5. Throw a fair die twice and let X be the outcome of first throw and let Y be the out come of second throw. Let $\mathcal{R}_1, \mathcal{R}_2$ be a rectangles whose sides have length X, X and X, Y respectively. Let A_1, A_2 be perimeters of $\mathcal{R}_1, \mathcal{R}_2$. Find probability distributions of A_1, A_2 [15]

6. There are 35 students in a class with distinct names. Name tags with these names are distributed randomly to these students. Let C be the number of students who got their own tag. Compute the expected value of C. (Hint: Use indicator functions and linearity of expectation.) [15]

7. Let $\{A_1, A_2, \ldots, A_n\}$ be events in a probability space $(\Omega, \mathcal{F}, P)$. Show that

$$P(\bigcap_{i=1}^{n} P(A_i)) \geq \sum_{i=1}^{n} P(A_i) - n + 1.$$

[10]